Journal of Applied Intelligent System (e-ISSN : 2502-9401 | p-ISSN : 2503-0493)
Vol. 5 No. 2, pp. 80 – 90

Whale Optimization Algorithm Bat Chaotic Map Multi Frequency for Finding Optimum Value

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Abstract - Optimization is one of the most interesting things in life. Metaheuristic is a method of optimization that tries to balance randomization and local search. Whale Optimization Algorithm (WOA) is a metaheuristic method that is inspired by the hunting behavior of humpback whales. WOA is very competitive compared to other metaheuristic algorithms, but WOA is easily trapped in a local optimum due to the use of encircling mechanism in its search space resulting in low performance. In this research, the WOA algorithm is combined with the BAT chaotic map multi-frequency (BCM) algorithm. This method is done by inserting the BCM algorithm in the WOA search phase. The experiment was carried out with 23 benchmarks test functions which were run 30 times continuously with the help of Matlab R2012a. The experimental results show that the WOABCM algorithm is able to outperform the WOA and WOABAT algorithms in most of the benchmark test functions. The increase of performance in the average of optimum value of WOABCM when compared to WOA is 2.27x10 ^ 3.

Keywords - : Optimization, Benchmark, Algorithm, Metaheuristic, Randomization

1. INTRODUCTION

Whale Optimization Algorithm (WOA) is a metaheuristic method inspired by the social behavior of the humpback whale in hunting its prey. This method is proposed in previous research [1] and has been tested with 29 mathematical optimization problems and 6 structured optimization problems. The test results show that WOA is very competitive when compared to optimization methods in the state of the art. WOA can be used to solve problems in various fields [2], including in electrical engineering, computer engineering, applied mathematics, and construction.

Regarding the WOA algorithm, many studies have been carried out, both by modifying WOA and combining WOA with other algorithms. Among the modifications to the WOA algorithm are AWOA, IWOA, Chaotic WOA, ILWOA and MWOA. As for what is included in the WOA hybridization is WOA combined with SA, PSO, LS, EWGC and BS-WOA. AWOA is a modification of WOA which in randomization uses adaptive techniques. This technique is very important in reducing the computation time of very complex problems [3]. IWOA was proposed [4] to correct the shortcomings of WOA which is unable to adapt to the nonlinear and complex WOA search process due to the presence of parameter a which decreases linearly from 2 to 0 during iteration. The experimental results show IWOA is more efficient than the original WOA in terms of convergence performance. Chaotic WOA was proposed [5] by using chaos to increase the convergence speed and performance of WOA. SA-WOA proposed by [6] is a combination of SA and WOA where SA is embedded into WOA to improve the best solution. SA is used to improve the exploitation phase and overcome stagnation in local optima. PSO-WOA was proposed by [7] to obtain a superior solution where PSO is used for the exploitation phase and WOA for the exploration phase. The results show that PSO-WOA is superior to PSO and WOA individually. BS-WOA [8] is a hybrid algorithm based on brainstorm optimization and WOA. BS-WOA is used to identify secret database keys.

Although the WOA algorithm is very competitive [1] compared to other optimization algorithms, WOA is easily trapped in a local optimum [9], [10]. This is due [1] to the use of encircling mechanism in the search space resulting in low performance. Like the WOA algorithm, the BAT algorithm [11] is also easily trapped in local optimum conditions so that it cannot perform global search properly. Improvements to the BAT algorithm [12] are proven to be able to improve the performance of the BAT algorithm in achieving the optimum value. The research problem arises from the methods mentioned above is the WOA algorithm combined with the multi-frequency Bat Chaotic Map (BCM) algorithm, which can be called WOABCM. This approach is carried out by inserting the BCM algorithm into the WOA search phase without modifying parameter a (WOABCM) or by modifying parameter a (WOABCM non-linear). This section discusses related research so that the contribution and position of the research carried out is clear.

Mirjalili [1] proposed a metaheuristic algorithm named Whale Optimization Algorithm (WOA) in 2016. This algorithm was inspired by the social behavior of humpback whales with a bubble net hunting strategy. This algorithm consists of two main stages, namely: the exploitation phase and the exploration phase. In the exploitation phase, encircling mechanism and spiral updating position are implemented. In the exploration phase, the search for prey is carried out randomly. In each iteration, each search agent updates its position based on the best solution when | A | <1 or based on the search agent randomly selected when | A |> 1. In order to obtain the exploration and exploitation phase, the value of the parameter a is decreased from 2 to 0 linearly during iteration. WOA has a parameter p (random value between 0 and 1) for determining circular or spiral motion. If the p value is greater than 0.5, the search agent changes its position using a spiral mechanism and if the value is less than 0.5 using a circular or random movement. The WOA algorithm was tested using 29 mathematical optimization functions and 6 structural design problems. The test results are compared with other metaheuristic optimization algorithms, such as: PSO, GSA, DE and others. The conclusion from their research results is that the WOA algorithm is very competitive [1] when compared to other metaheuristic algorithms and conventional methods.

Although the WOA algorithm is very competitive, it is linear and cannot adapt to the non-linear and complex WOA search process [4]. In order to adapt to the non-linear search process, [4] proposed several strategies in regulating distance control parameters. The proposed algorithm is known as IWOA. There are five types of IWOA based on distance control parameter variables, namely: SinWOA, CosWOA, TanWOA, LogWOA and SquareWOA. The experimental results on the 6 benchmark test functions show that the nonlinear a distance control parameter strategy is superior to the classic WOA algorithm linear control strategy. Apart from parameter modification, the WOA algorithm has also been combined with other metaheuristic algorithms. One of them is the WOABAT algorithm. The WOABAT algorithm [13] is a combination of the WOA algorithm and the BAT algorithm. In testing, the BAT algorithm is used for the exploration phase. The WOA algorithm is used for the exploitation phase. The BAT algorithm is inserted into the search phase for the WOA algorithm. The a parameter setting is the same as the original WOA which decreases linearly from 2 to 0. In the experiment [13] tested with 25 mathematical functions and compared with the WOA algorithm. The comparison results show that the WOABAT algorithm outperforms the WOA algorithm in 13 mathematical functions. WOABAT also has a lower average optimum value than WOA in 7 out of 10 mathematical functions.

The BAT algorithm is a metaheuristic algorithm that mimics the behavior of groups of bats in hunting prey, avoiding obstacles and finding nests located far away or in dark conditions, proposed by [13]. This algorithm is divided into several parts, namely: initialization, updating of frequency, speed and position, local search and updating of pulse emission and sonar wave amplitude. Four benchmark test functions are used and then compared with several other metaheuristic algorithms. In conclusion, it is stated that the BAT algorithm is potentially much more reliable than GA, PSO and HS. The BAT algorithm has a weakness, which is that it is easily trapped in a local optimum. [12] proposed an improvement to the BAT algorithm using chaotic maps and multi-frequency, hereinafter called the BCM algorithm. In this algorithm, multi-frequency parameters are used to improve exploration capabilities. The chaotic map is used to improve exploitation capabilities.

2. RESEARCH METHOD

A group of whales in hunting for prey is represented by a matrix. The matrix measures (i, dim) where i is the number of finding agents (whales) and dim is the number of whale dimensions. A random method was used to form a matrix of the whales' positions. Furthermore, the matrix formed is used as input in the WOABCM algorithm. In this study, the WOA algorithm without modification of parameter a or modification of parameter a is combined with the multi-frequency chaotic map BAT algorithm. Modifications are made by changing the strategy of the distance control parameter a to be non-linear. The algorithm is combined with the BAT chaotic map multi-frequency algorithm to perform global searches (exploration phase) as well as local searches that replace the encircling mechanism. The purpose of modification and hybridization is to improve the performance of the WOA algorithm. To measure how much the WOABCM algorithm has improved, a statistical evaluation method is used. The statistical evaluation method consists of the best optimum value, average value and standard deviation.

The data that will be tested in this study are in the form of a mathematical test function (benchmark function) obtained from previous research [1]. These data are commonly used by researchers to test the performance of an optimization algorithm. Many researchers have used this test function, including [1], [17], [18], [19].

2.1. The method proposed

In this study, an experiment was conducted on the application of the WOABCM algorithm, a WOA algorithm, which is inserted in the exploration phase and part of the exploitation phase with the improved BAT algorithm with chotic maps and multi-frequency. This is because [13] because the WOA algorithm (circular mechanism) has a lower ability to free itself from local optima. The results of [13] show that the combination of the WOA algorithm and the BAT algorithm (WOABAT) is better than the WOA algorithm with fewer iterations. The result of research [12] that the improved BAT algorithm with chaotic map and multi-frequency (BCM) is able to free itself from local optima, thereby increasing the performance of the BAT algorithm. By combining the WOA algorithm with the BCM algorithm, it can increase population diversity and is able to avoid trapping the local optimum, thereby increasing the performance of the WOA algorithm in general in reaching the global optimum point or closer to it.

This algorithm begins with initialization of the whale / hunting agent population represented in the form of a matrix. Each one-dimensional array representing each search agent contains the position value of a randomly generated search agent. At initialization, each whale position is evaluated using the objective function to get the best function value and the best whale position. Next is the iteration process where in this process there is an update of several parameters a, A, C, l and p. These parameters affect the next stage, both exploration and exploitation. In this process, there is a sharp difference between the WOA algorithm and the proposed algorithm. In the WOA algorithm, parameter a is derived linearly from 2 to 0 during iteration. As for the proposed algorithm parameter a is regulated either linearly or non-linearly. In this proposed algorithm, the non-linear distance control parameter strategy used is equation (2), (3), (4), and (5). This equation replaces equation (1) which is the linear distance control parameter strategy found in the WOA algorithm.

$$a(t) = 2 - 2\frac{t}{t_{mak}} \tag{1}$$

$$a(t) = (a_{mak} - a_{min})x \sin\left(mu.\frac{t}{t_{mak}}.pi\right) \tag{2}$$

$$a(t) = (a_{mak} - a_{min})x \cos\left(mu.\frac{t}{t_{mak}}.pi\right) \tag{3}$$

$$a(t) = (a_{mak} - a_{min})x \tan\left(mu.\frac{t}{t_{mak}}.pi\right) \tag{4}$$

$$a(t) = (a_{mak} - a_{min})x \left(\frac{t}{t_{mak}}\right)^2 \tag{5}$$

In the original WOA algorithm, the encircling mechanism is still used. As for the proposed algorithm, the encircling mechanism is no longer used. Instead, the BCM algorithm is used. In each iteration, each search agent updates its position using one of two conditions. If | A | less than 1 position renewal process based on the best solution achieved to date. If | A | greater than 1 position renewal process based on randomly selected search agents.

There are two types of movement in the proposed WOA algorithm, namely: BCM algorithm movement and spiral movement. Another parameter namely p (random number in the range between 0 and 1) determines the movement. If the p value is less than 0.5 the movement follows the BCM algorithm, in this case equations (6) to (12) are used.

$$Qs_i = Q_i * (1 + Sri * \frac{(x_{acak} - \mathrm{x}_*^t)}{|x_{acak} - \mathrm{x}_*^t| + realmin}) \tag{6}$$

$$Qs_i = Q_i * (1 + Sri * \frac{(\mathrm{x}_i^t - \mathrm{x}_*^t)}{|\mathrm{x}_i^t - \mathrm{x}_*^t| + realmin}) \tag{7}$$

$$v_i^t = w + v_i^{t-1} + (\mathrm{x}_i^t - \mathrm{x}_*^t) * Qs_i \tag{8}$$

$$z_i^t = z_i^{t-1} + v_i^t \tag{9}$$

$$X_{k+1} = \cos\left(a * \cos^{-1}(X_k)\right) \tag{10}$$

$$\epsilon = chaos(t) * \left|Amp_i^t - Amp_{mean}^t\right| + \varepsilon \tag{11}$$

$$z_i^t = \mathrm{x}_*^t * (1 + \epsilon) \tag{12}$$

Whereas if the p value is greater than 0.5, the spiral movement that applies to the WOA algorithm is used by following equations (13) and (14). Finally, the algorithm is completed when the termination conditions (maximum number of iterations) are reached.

$$D = |X^*(t) - X(t)| \tag{13}$$

$$X(t+1) = D'.e^{bl}.\cos(2.\pi.l) + X^*(t) \tag{14}$$

The process flow of the proposed method can be seen in Figure 1.

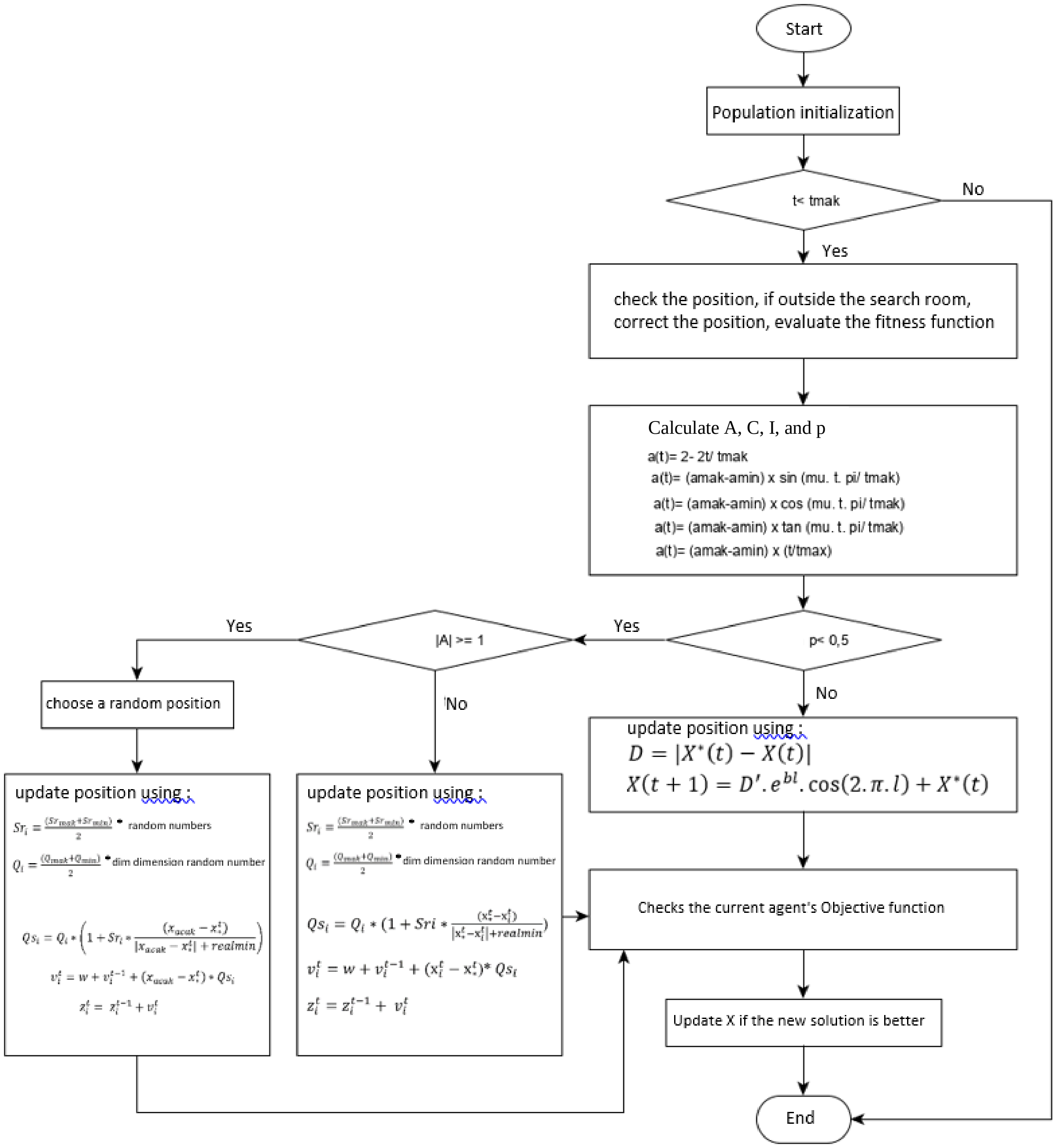

Figure 1. The Proposed Method

2.2. Stages of experiment

Experiments were carried out with the help of the Dell E6510 Laptop, the Matlab application version R2012a and Microsoft Excel 2013. In connection with achieving the optimum value of the 23 objective functions tested, this experiment was carried out in several stages, as follows:

1. Initialize a random search agent population.
2. Preparing 23 benchmark functions that are used to test the performance of the optimization algorithm.
3. Forming a WOA algorithm model.
   Research [1] is used as a reference in determining the parameters of the WOA algorithm. The WOA parameter settings are as follows: The number of search agent

population formed is 30 for each benchmark function. The maximum number of iterations for each benchmark function is 500 iterations. The number of dimensions is 30 for the F1 function or adjusted to the benchmark function used when the experiment was carried out. The parameter a is derived linearly from 2 to 0 during iteration. The determination of these parameters refers to previous studies [1].

4. Running the WOA algorithm testing process with 23 specified functions.
   Testing is carried out independently between one function and another and is carried out repeatedly as many as 30 experiments. The optimum value obtained from this experiment is the average value and standard deviation of this algorithm compared to other algorithms.
5. In the same way as steps 1,2 and 4, it is also used for the non-linear WOABAT, WOABCM, WOABCM algorithms. The difference is in stage 3 where the references used in setting different parameters. WOABAT refers to [13], WOABCM refers to [12], [13] and non-linear WOABCM refers to [4], [12], [13].
6. Comparing non-linear WOA, WOABAT, WOABCM, WOABCM algorithms, based on statistical evaluation (average optimum value and standard deviation).

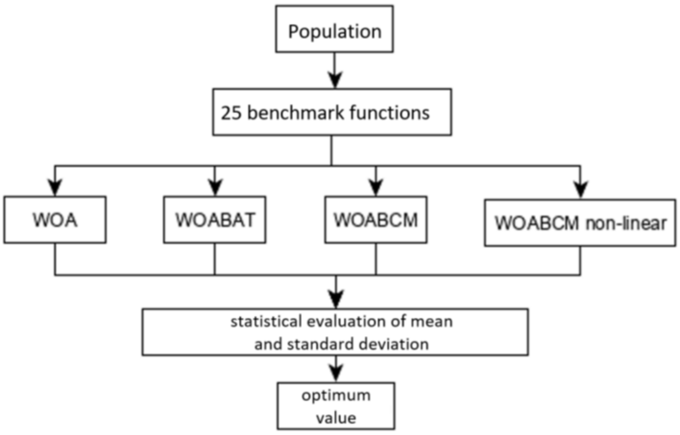

Figure 2. Stages of Experiment

Several previous researchers measured the optimum value using statistical evaluations: mean and standard deviation [1], mean and standard deviation [13], mean value, standard deviation and best value [4], average value mean, standard deviation, best value and worst value [12] from the optimum value achieved. In this study, statistical evaluation was carried out by measuring the components of the best value, the average value and the standard deviation. The main measurement components that will be used as a comparison are the average value and the standard deviation of the optimum value achieved. The algorithms that will be compared are WOA, WOABAT, WOABCM and non-linear WOABCM. If the average optimum value of the proposed algorithm is closer to the global optimum point of the tested function compared to the previous algorithms, it means that this research is able to contribute to knowledge, vice versa. The mean value (Ave) [9] is the solution value obtained from the results of M running, where Fi is the solution obtained from running it i and M is the number of programs executed, calculated by:

$$Ave = \frac{1}{M}\sum_{i=1}^{M} F_i \tag{15}$$

And the standard deviation (Std) [9] as an indicator of the divergence of a solution, is calculated based on the following equation:

$$Std = \sqrt{\frac{1}{M-1}\sum_{i=1}^{M}(F_i - Ave)^2} \tag{16}$$

3. RESULTS AND DISCUSSION

The results of the experiment are to find out how superior the performance of each optimization algorithm is, both the proposed optimization method and the algorithms used in the previous optimization. To determine the exploitation capability of each algorithm, several unimodal functions are used, namely a function that has only one local optimum (F1 to F7). Meanwhile, to determine the ability of exploration, several multimodal functions are used, namely functions that have several local optimals (F8 to F23).

Table 1.The experimental results of the WOA algorithm combined with the multi-frequency bat chaotic map algorithm (WOABCM)

Function Name	Global Optimum Point	WOABCM		
		The best value	Mean	Standard Deviation
F1	0	**0**	2,207E-187	0
F2	0	2,181E-258	2,4888E-84	1,3632E-83
F3	0	**0**	1,825E-22	9,85469E-22
F4	0	3,779E-252	9,7947E-96	5,36477E-95
F5	0	5,66E-16	5,2465E-08	1,59875E-07
F6	0	9,1794E-26	8,236E-09	4,49908E-08
F7	0	3,2143E-05	0,00087108	0,000985802
F8	-4,19E+02	-12569,4866	-12332,6099	901,4630616
F9	0	**0**	**0**	0
F10	0	8,8818E-16	8,8818E-16	0
F11	0	**0**	**0**	0
F12	0	2,1267E-24	1,8444E-11	4,14881E-11
F13	0	4,4446E-18	6,2626E-11	1,46593E-10
F14	1	0,99800384	1,35920961	1,380555259
F15	3,00E-04	0,00030749	0,00033057	8,45413E-05
F16	-1,03E+00	-1,03162845	-1,03162845	4,28502E-16
F17	3,98E-01	0,39788736	0,39788736	2,13556E-13
F18	3,00E+00	**3**	5,7	8,238471569
F19	-3,86E+00	-3,86278215	-3,81124762	0,196120949
F20	-3,32E+00	-3,32199509	-3,27745753	0,057767787
F21	-1,02E+01	-10,1531997	-9,81637118	1,281841951
F22	-1,04E+01	-10,4029406	-9,69835317	1,827073544
F23	-1,05E+01	-10,5364098	-10,1758812	1,372035571

Based on the experimental results as shown in Table 1, it is known that the WOA algorithm combined with the multi-frequency BAT chaotic map (BCM) algorithm or the so-called WOABCM can achieve global optimum values in several test functions. Of the 23 test functions, the WOABCM algorithm can reach the global optimum value when tested with 5 (five) test functions, in particular F1, F3, F9, F11 and F18. This can be seen in the numbers in bold in the best value column. The optimum value achieved by the WOABCM algorithm can be seen from the average value when tested with the F9 and F11 functions (you can see the numbers in bold in the average column) where the test is repeated 30 times. When viewed from the standard deviation, 4 benchmark test functions with a value of 0 (on the F1, F9, F10, and F11 functions) the WOABCM algorithm can achieve. For other benchmark functions, the optimum value of this proposed algorithm is also not far from the global optimum value of each of the benchmark test functions.

To find out how superior the proposed algorithm is when compared to the previous algorithms, in the next section a comparison is made between the WOA algorithm and the WOABCM algorithm and a comparison between the WOABAT algorithm and the WOABCM algorithm and the comparison between CosWOABAT, SinWOABAT, TanWOABAT, SquareWOABAT and WOABCM.

Table 2. Comparison of the optimal value of the WOA algorithm with WOABCM

Function Name	Global Optimum Point	WOA		WOABCM	
		Mean	Standard Deviation	Mean	Standard Deviation
F1	0	1,60E-73	6,20E-73	**2,207E-187**	**0**
F2	0	2,37E-51	8,82E-51	**2,4888E-84**	**1,3632E-83**
F3	0	5,22E+04	15011,56407	**1,825E-22**	**9,85469E-22**
F4	0	4,60E+01	25,29754414	**9,7947E-96**	**5,36477E-95**
F5	0	2,79E+01	0,447957398	**5,2465E-08**	**1,59875E-07**
F6	0	4,34E-01	0,235396645	**8,236E-09**	**4,49908E-08**
F7	0	3,94E-03	0,003865539	**0,00087108**	**0,000985802**
F8	-4,19E+02	**-1,03E+04**	1798,374488	-12332,6099	**901,4630616**
F9	0	0	0	0	0
F10	0	4,44E-15	2,29E-15	**8,8818E-16**	**0**
F11	0	0	0	0	0
F12	0	2,81E-02	0,029928715	**1,8444E-11**	**4,14881E-11**
F13	0	5,56E-01	0,252276153	**6,2626E-11**	**1,46593E-10**
F14	1	3,28E+00	3,874643654	**1,35920961**	**1,380555259**
F15	3,00E-04	6,42E-04	0,000405665	**0,00033057**	**8,45413E-05**
F16	-1,03E+00	-1,03162845	2,79479E-09	-1,03162845	**4,28502E-16**
F17	3,98E-01	0,397897047	1,58508E-05	**0,39788736**	**2,13556E-13**
F18	3,00E+00	**3,000096042**	**0,000181237**	5,7	8,238471569
F19	-3,86E+00	**-3,85601284**	**0,009501102**	-3,81124762	0,196120949
F20	-3,32E+00	-3,2132765	0,161430485	**-3,27745753**	**0,057767787**
F21	-1,02E+01	-8,43036078	2,460341102	**-9,81637118**	**1,281841951**
F22	-1,04E+01	-7,90646129	3,180296493	**-9,69835317**	**1,827073544**
F23	-1,05E+01	-7,97801572	3,502467061	**-10,1758812**	**1,372035571**

Based on Table 2, it is clear that WOABCM is able to outperform the WOA algorithm in most of the test functions. The superior performance of the WOABCM algorithm can be seen from the optimum average value and standard deviation value of the algorithm. Seen in the WOABCM algorithm, most of the average value of each test function obtained is closer to the global optimum point than the WOA algorithm. Of the 23 test functions, 17 test functions in the WOABCM algorithm are closer to the global optimum point than the WOA algorithm (can be seen in bold numbers in the WOABCM mean value column). For the standard deviation value, the WOABCM algorithm is also mostly better than the WOA algorithm. This can be seen in the closer the standard deviation value of the WOABCM algorithm to zero, even in several test functions such as F1, F9, F10 and F11 the standard deviation value is zero.

Table 3. Comparison of the optimal value of the WOABAT algorithm with WOABCM

Function Name	Global Optimum Point	WOABAT		WOABCM	
		Mean	Standard Deviation	Mean	Standard Deviation
F1	0	1,34E-06	4,07E-07	**2,207E-187**	**0**
F2	0	7,43E-03	0,001299535	**2,48884E-84**	**1,3632E-83**
F3	0	9,62E-06	1,70E-06	**1,82502E-22**	**9,85469E-22**
F4	0	1,02E-03	7,76E-05	**9,79468E-96**	**5,36477E-95**
F5	0	7,48E+00	12,61254018	**5,24649E-08**	**1,59875E-07**
F6	0	1,59E-06	5,74E-07	**8,23596E-09**	**4,49908E-08**
F7	0	1,40E-03	0,002770249	**0,000871079**	**0,000985802**
F8	-4,19E+02	-1,25E+04	**648,7134684**	**-12332,6099**	901,4630616
F9	0	3,98E+00	10,32008077	**0**	**0**
F10	0	9,10E-04	0,000216198	**8,88178E-16**	**0**
F11	0	8,25E-08	3,61E-08	**0**	**0**
F12	0	1,60E-08	6,42E-09	**1,84443E-11**	**4,14881E-11**
F13	0	2,25E-07	1,03E-07	**6,26256E-11**	**1,46593E-10**
F14	1	2,75E+00	4,081160311	**1,359209609**	**1,380555259**
F15	3,00E-04	3,76E-04	0,000335943	**0,000330566**	**8,45413E-05**
F16	-1,03E+00	-1,0316285	6,28037E-16	**-1,03162845**	**4,28502E-16**
F17	3,98E-01	**0,3978874**	**6,17936E-14**	0,397887358	2,13556E-13
F18	3,00E+00	10,2	12,14396403	**5,7**	**8,238471569**
F19	-3,86E+00	**-3,8627821**	**1,42663E-12**	-3,81124762	0,196120949
F20	-3,32E+00	**-3,2937062**	**0,059743772**	-3,27745753	0,057767787
F21	-1,02E+01	-8,9682233	2,184699927	**-9,81637118**	**1,281841951**
F22	-1,04E+01	-9,5211775	2,005405947	**-9,69835317**	**1,827073544**
F23	-1,05E+01	-9,2792763	2,317732154	**-10,1758812**	**1,372035571**

Based on Table 3 above, it is clear that the performance of the WOABCM algorithm is superior to the WOABAT algorithm in most of the test functions. Seen in the WOABCM algorithm, the average value of each test function is mostly closer to the global optimum point of each test function than the WOABAT algorithm (out of 23 test functions there are 20 test functions with an average optimum value closer to the global optimum point.). Even in the test functions F9 and F11 the WOABCM algorithm is able to reach the global optimum value of the function. For the standard deviation value, the WOABCM algorithm is also mostly better than the WOA algorithm. The standard deviation value of the WOABCM algorithm is getting closer to zero, even in some test functions F1, F9, F10 and F11 the standard deviation value is zero.

Table 4. Comparison of the Optimum Value of the CosWOABCM, SinWOABCM, TanWOABCM, SquareWOABCM algorithms with WOABCM

Fuction Name	Global Optimum Point	CosWOABCM		SinWOABCM		TanWOABCM		SquareWOABCM		WOABCM	
		Mean	Std.dev.	Mean	Std.dev.	Mean	Std.dev.	Mean	Std.dev.	Mean	Std.dev.
F1	0	2,4E-146	1,3E-145	3,3E-103	1,8E-102	9,1E-79	4,99E-78	3,16E-59	1,73E-58	**2,2E-187**	**0**
F2	0	1,69E-32	9,28E-32	1,58E-46	8,67E-46	5,66E-52	3,1E-51	1,64E-29	8,98E-29	**2,49E-84**	**1,36E-83**
F3	0	2,17E-12	1,19E-11	2,39E-45	1,22E-44	**2,06E-65**	**1,13E-64**	5,9E-44	3,23E-43	1,83E-22	9,85E-22
F4	0	5,22E-40	2,86E-39	2,83E-12	1,55E-11	3,67E-21	2,01E-20	8,77E-11	4,8E-10	**9,79E-96**	**5,36E-95**
F5	0	6,07E-07	1,77E-06	1,78E-07	6,51E-07	9,72E-08	2,33E-07	9,16E-07	2,51E-06	**5,25E-08**	**1,6E-07**
F6	0	**3,67E-09**	1,65E-08	2,85E-09	**8,69E-09**	5,77E-08	3,1E-07	7,93E-09	3,15E-08	8,24E-09	4,5E-08
F7	0	0,000432	0,000469	0,000468	0,000516	**0,00036**	**0,000337**	0,000565	0,0004	0,000871	0,000986
F8	-4,19E+02	-12569,5	**0,000571**	-12569,5	0,007648	-12569,5	0,001076	-12460,1	509,8822	**-12332,6**	901,4631
F9	0	0	0	0	0	0	0	0	0	**0**	**0**
F10	0	2,93E-11	1,61E-10	8,88E-16	0	4,46E-10	2,44E-09	8,88E-16	0	**8,88E-16**	**0**
F11	0	0	0	0	0	3,94E-11	2,16E-10	2,59E-17	1,42E-16	**0**	**0**
F12	0	1,73E-10	8,52E-10	1,44E-10	3,33E-10	3,76E-10	1,51E-09	6,68E-10	1,72E-09	**1,84E-11**	**4,15E-11**
F13	0	3,38E-09	8,74E-09	2,97E-08	1,47E-07	2,76E-08	1,35E-07	1,68E-09	2,89E-09	**6,26E-11**	**1,47E-10**
F14	1	**0,998004**	**2,78E-16**	**0,998004**	3,49E-16	**0,998004**	3,44E-16	**0,998004**	3,58E-16	1,35921	1,380555
F15	3,00E-04	**0,000308**	**3,29E-06**	0,001783	0,005368	0,00071	0,001497	0,001756	0,005366	0,000331	8,45E-05
F16	-1,03E+00	-1,03163	5,89E-16	-1,03163	2,35E-15	-1,03163	6,1E-16	-1,03163	5E-16	**-1,03163**	**4,29E-16**
F17	3,98E-01	0,397887	7,51E-08	0,397887	1,56E-08	0,397887	6,68E-09	0,397887	5,62E-09	0,397887	2,14E-13
F18	3,00E+00	7,761054	17,28727	3,9	4,929503	**3**	1,74E-14	**3**	7,29E-15	5,7	8,238472
F19	-3,86E+00	-3,81637	0,120362	**-3,84731**	0,065438	-3,83701	0,141133	-3,86223	**0,001998**	-3,81125	0,196121
F20	-3,32E+00	**-3,29382**	0,061041	-3,27007	0,081977	-3,27839	0,06614	-3,24607	0,075636	-3,27746	**0,057768**
F21	-1,02E+01	-9,98479	0,922443	**-10,1532**	**1,16E-09**	**-10,1532**	2,15E-05	**-10,1532**	4,25E-05	-9,81637	1,281842
F22	-1,04E+01	**-10,4029**	7,69E-11	**-10,4029**	**2,02E-11**	**-10,4029**	2,1E-11	-10,4021	0,00461	-9,69835	1,827074
F23	-1,05E+01	**-10,5364**	5,97E-11	**-10,5364**	1,16E-10	**-10,5364**	**2,04E-11**	-10,3577	0,978737	-10,1759	1,372036

Based on Table 4, the performance of the WOABCM algorithm is superior to the CosWOABCM, SinWOABCM, TanWOABCM, SquareWOABCM and WOA-BCM algorithms in most test functions. Seen in the WOABCM algorithm, some of the average value of each test function approaches the global optimum value of each of these test functions (of 23 test functions, there are 9 test functions that show the advantages of WOABCM). Even in the test functions F9 and F11 this algorithm is able to reach the global optimum value of the function. For the standard deviation value on the WOABCM algorithm, most of them are better than the CosWOABCM algorithm, SinWOABCM, TanWOABCM, SquareWOABCM. For CosWOABCM excels in 6 test functions, namely: F6, F14, F15, F20, F22 and F23. For SinWOABCM it excels in 5 test functions, namely: F14, F19, F21, F22 and F23. For TanWOABCM it excels in 6 test functions, namely: F3, F7, F14, F21, F22 and F23. As for SquareWOABCM it excels in 3 test functions, namely: F14, F18 and F21. The standard deviation value of the WOABCM algorithm is getting closer to zero, even in some test functions F1, F9, F10 and F11 the standard deviation value is zero.

Table 5. Performance of the WOABCM algorithm

Fuction Name	Global Optimum Point	WOA	WOABCM	Performance improvements
		mean	mean	
F1	0	1,60E-73	**2,2E-187**	1,60E-73
F2	0	2,37E-51	**2,49E-84**	2,37E-51

F3	0	5,22E+04	**1,83E-22**	**5,22E+04**
F4	0	4,60E+01	**9,79E-96**	4,60E+01
F5	0	2,79E+01	**5,25E-08**	2,79E+01
F6	0	4,34E-01	**8,24E-09**	4,34E-01
F7	0	3,94E-03	**0,000871**	3,07E-03
F8	-4,19E+02	-1,03E+04	-12332,6	0,00E+00
F9	0	0	**0**	0,00E+00
F10	0	4,44E-15	**8,88E-16**	3,55E-15
F11	0	0	**0**	0,00E+00
F12	0	2,81E-02	**1,84E-11**	2,81E-02
F13	0	5,56E-01	**6,26E-11**	5,56E-01
F14	1	3,28E+00	**1,35921**	1,92E+00
F15	3,00E-04	6,42E-04	**0,000331**	3,11E-04
F16	-1,03E+00	-1,03163	-1,03163	0,00E+00
F17	3,98E-01	0,397897	0,397887	0,00E+00
F18	3,00E+00	3,000096	5,7	0,00E+00
F19	-3,86E+00	-3,85601	**-3,81125**	-4,48E-02
F20	-3,32E+00	-3,21328	**-3,27746**	6,42E-02
F21	-1,02E+01	-8,43036	**-9,81637**	1,39E+00
F22	-1,04E+01	-7,90646	**-9,69835**	1,79E+00
F23	-1,05E+01	-7,97802	**-10,1759**	2,20E+00
average performance				**2,27E+03**

Of the 23 functions in Table 5 above, there are 17 functions that have improved algorithm performance. The biggest performance improvement is in the F3 function. The increase in the average performance of the WOABCM algorithm is 2.27x10 ^ 3.

4. CONCLUSION

The WOABCM algorithm is able to outperform the original WOA algorithm and WOABAT. This can be proven after being tested with 23 benchmark test functions. The increase in the average performance of the WOABCM algorithm is 2.27x10 ^ 3. WOABCM also has good capabilities in terms of exploitation and exploration. Therefore, for future research, further research can be carried out on the application of WOABCM related to optimization problems in other fields such as data mining, electrical engineering, civil engineering, mechanical engineering and others.

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